We present the Third Palermo Swift-BAT hard X-ray catalog obtained by analyzing data acquired in the first 66 months of the Swift mission. Using our software dedicated to the analysis of data from coded mask telescopes, we analyzed the BAT survey data in different energy bands between 15 and 150 keV. The survey covers 50% of the sky to a 15-150 keV flux limit of $7.5 \times 10^{-12}$ erg cm$^{-2}$ s$^{-1}$. The source detection algorithm has been improved to optimize the energy band and the time interval that allow to achieve the maximum signal to noise ratio for each pixel of the all-sky map. We obtain a list of $\sim 1700$ source candidates with a negligible number of spurious detections. The identification of the source counterparts in the softer energy band is pursued using two strategies: the analysis of field observations of soft X-ray instruments and cross-correlation of our catalog with source databases. The distribution of the BAT sources among the different object classes consists of $\sim 15\%$ Galactic sources, $\sim 49\%$ extragalactic sources, $\sim 12\%$ sources with a counterpart at softer energies whose nature has not yet been determined while $\sim 24\%$ still lack any associated counterpart.